

Safety and Operational Performance of Autonomous Vehicles at Roundabouts

INTRODUCTION

Project Title: Safety and Operational Performance of Autonomous Vehicles at Roundabouts

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Summary Table	
Project Name/Title	Safety and Operational Performance of Autonomous Vehicles at Roundabouts
Eligible Entity Applying to Receive Federal Funding (Prime Applicant's Legal Name and Address)	Rutgers University – Camden 303 Cooper Street, Camden, NJ 08102
Point of Contact (Name/Title; Email; Phone Number)	Benedetto Piccoli, Distinguished Professor; piccoli@camden.rutgers.edu ; 856-225-6356
Proposed Location (State(s) and Municipalities) for the Demonstration	Delaware, New Jersey, Rhode Island
Proposed Technologies for the Demonstration (briefly list)	Roadside sensing and cameras, on-board sensors
Proposed duration of the Demonstration (period of performance)	3 Years (July 2019 to June 2022)
Federal Funding Amount Requested	\$5,918,880

Cover Letter

To the US DOT.

Please accept the present application to the NOFO Number 693JJ319NF00001 "Automated Driving System Demonstration Grants".

The proposed project will realize a unique multi-level testbed for safety of Automated Driving System at roundabouts with potential industrial impact at general intersections in urban and rural environment.

Site will be selected presenting roundabouts with one and two lanes. Then virtual and artificial environments will be tuned to the selected sites and firstly used to generate a test-beds for ADS immersed in traffic and alternative control policies.

Then, using sideroad and on-board technologies, the project will collect and render available data corresponding to more than 5,000 hours and 100,000 miles of driving at roundabouts and intersections.

The project will have performance sites in five states: Alabama, Delaware, Idaho, New Jersey and Rhode Island, ensuring a span of different driving behavior, weather and environment conditions.

The project PIs: Piccoli, Chang, Hammond, Hunter, Malikopoulos

Table of Contents

EXECUTIVE SUMMARY	1
BACKGROUND.....	2
FOCUS AREAS	3
A. Significant Public Benefits	4
B. Addressing Market Failure and Other Compelling Public Needs	5
C. Economic Vitality	6
D. Complexity of Technology.....	7
E. Diversity of Project.....	7
F. Transportation-challenged Populations.....	7
G. Prototypes	7
REQUIREMENTS	8
APPROACH	8
Task 1: Equip both single and double-lane roundabouts with cameras and sensors	9
Apponaug Circulator	10
Measuring Traffic Stream Parameters during Demonstration	11
Task 2: Develop a software suite including microscopic and macroscopic models calibrated for the sites	12
Task 3: Extend and calibrate the mini-city environment for the sites, including matching software outcomes	14
Task 4: Design control algorithms for braking events and congestion reduction using pipeline of tasks 1-3 during merging operation.....	17
Task 5: Use AVs and ADS-equipped cars to test control algorithms and currently available technology at roundabouts and junctions in five states	18
Task 6: Management.....	21
REFERENCES CITED.....	23

LIST OF FIGURES

Figure 1: Apponaug Circulator, Study Sites Highlighted (Source: RIDOT)	9
Figure 2: Double Lane Roundabout at Veterans Memorial Drive and Post Road (Source: RIDOT)	10
Figure 3: Single-Lane Roundabout at Centerville Road and Greenwich Road (Source: RIDOT)	11
Figure 4: A road network representing a roundabout.....	13
Figure 5: A birds-eye view of the University of Delaware's Scaled Smart City.	15
Figure 6: Vehicles used in the scaled smart city with internals (left) and outer shell (right).	16
Figure 7: Controlled AV eliminating traffic waves and braking events.....	17
Figure 8: Speed profile (left) and conflict heat-map (right) (Source: BriskSynergies [19]).....	19
Figure 9: Demo Vehicle Audi 3.....	20

EXECUTIVE SUMMARY

Autonomous vehicles (AVs) will soon form a larger part of our transportation system due to consistent advancements in driverless technology. Automakers are already testing the technology in Arizona, California, Pennsylvania and other states across the US. One of the main benefits that is often promoted with AVs is safety, which is very true, but there are still unanswered questions as to how AVs can be integrated in existing transportation infrastructure without compromising its safety benefits.

One particular area that needs further attention is AV operation at locations such as roundabouts, merges, unprotected left turns, and stop signs which require a significant amount of human input to navigate. This is also evident in the ongoing Arizona pilot program launched by Waymo, Google's self-driving car unit, where human drivers had encountered that self-driving cars had trouble merging onto freeways and also navigating unprotected left turns. Several studies have proposed different algorithms to address AV merges but the lack of funds and readily available data from both deployed AVs and theoretical models makes it difficult to ascertain whether proposed methods can safely execute merging operation with human drivers.

As the technology advances and government officials develop guidelines for AV operation, it is important that we examine how to safely integrate AVs into our transportation infrastructure systems with main focus on infrastructure that were designed mainly for human driver operations. This project focuses on the underlying technologies required to investigate the safety and traffic operational implications of various headway acceptances of an AV at roundabouts. Using a developed algorithm for predicting braking events, analysis would be performed for various critical headway acceptances of an AV on an approach of a roundabout so as to assess its effect on the safety of vehicles on the circulatory pathway. Throughput and speeds are the other measure effectiveness that would be studied. The proposed project would use selected single and double lane roundabout theoretical modeling and physical demonstration over a three year period. Project data will be collected in data repositories online and shared with the USDOT.

This collaborative project involves expertise in Automated Driving System (ADS), traffic modeling and control from the University of Delaware, Rutgers University, University of Alabama at Birmingham, University of Idaho, and the University of Rhode Island. State and local governments where these institutions and demonstration site are situated are also involved in this project as they seek knowledge for policymaking. The project team combines expertise in traffic modeling, roundabouts, distributed sensing, estimation, control, software, and experimental aspects of AVs. However, each of these areas are limited in their contribution to the research agenda in this proposal if considered separately. The timescales and dynamics of the controllers needed to dampen the traffic events are sufficiently short that traditional approaches to control and estimation might exacerbate, rather than dissipate, congestion events. In order to estimate the traffic state, and in particular non-local network effects,

computationally trackable in real-time, mathematically complex micro-macro models are required.

This project provides the mathematical, computational, and engineering structure necessary for integrating human drivers with current and future civil infrastructure systems and possibly reap the environmental and economic benefits associated with these systems. The results from this research will significantly advance knowledge and understanding on the safety and operational impact of integrating AVs with human drivers at roundabouts and other transportation infrastructure designed to operate in a similar manner. This critical knowledge will be necessary for designing transportation infrastructure that can safely accommodate AV and developing guidelines for AV operation with human drivers. Furthermore, this knowledge will be important for developing new AV algorithms and technologies that can safely operate with human drivers at roundabouts and other intersections that require significant human input to operate.

BACKGROUND

Over 30,000 people lose their lives in the US annually in motor vehicle crashes and 90 percent of these fatal crashes are caused by driver errors [1]. Motor vehicle crashes as a whole can pose economic and social costs of more than \$800 billion in a single year [2]. Autonomous vehicles (AVs) have the potential to minimize these crashes as they could eliminate most of the mistakes made by human drivers.

AVs operate without direct driver input to control the steering, acceleration, and braking. They utilize computerized systems to detect and collect information about the environment, identify paths and hazards, as well as control functions such as acceleration and steering, to navigate the vehicle accordingly [3]. While this technology makes driving safer and efficient, it also poses some challenges for certain transportation infrastructures that require human decision making to navigate. For instance, operations at roundabouts as opposed to regular signalized intersections are strongly affected by driver behavior where a driver has to scan for a sufficient gap to enter the circulatory pathway. This involves constant braking, accelerating, decelerating and good-judgment decision on the part of the driver. More importantly, drivers use hand gestures or eye contact to express their intentions to merge into the circulatory pathway during congested periods. These driver inputs which AVs find to be very challenging are critical for the safe operation of roundabouts.

This work is a departure from previous investigations on autonomous vehicles, which have largely focused on a) the control of a single AV immersed within human drivers with the goal to optimize the AV itself [4, 5, 6, 7]; or b) a roadway with only AVs on it, and suitable controls of such a fully coupled AV fleet [8, 9, 10, 11]. In contrast, the new perspectives in this project are: 1) AVs immersed within human drivers including older and disabled drivers; and 2) AVs used to optimize properties of the bulk, human controlled traffic flow.

Examining the safety requirements for integrating ADS technology into our transportation system could be very challenging as there are so many parameters in addition to human drivers to consider. There is also the issue of performing a study that can be applied to as many different transportation components as possible. The main issue with ADS technology is with merging onto freeways and also navigating unprotected left turns. But the question is how one can study these operations without minimum risk? If tested at freeway influence areas, there is the issue with speed of vehicle on the freeway which makes the study dangerous and unsafe. The best approach to examining the main issue facing ADS technology is with the use of roundabouts. Roundabouts have fewer conflict points than a conventional intersection. A conventional intersection has 32 conflict and a roundabout has only eight. Roundabouts also operate at lower speed, typically around 25 mph. Roundabout are also capable of accommodating different road users such pedestrians, bikes, large trucks and people with disabilities. More importantly, a roundabout uses merging operation where entering vehicles need to yield and enter the circle when there is a gap of acceptable length.

This makes the roundabout an ideal candidate for examining the safety performance of ADS integration into transportation system as it is much safer, has fewer conflict points, poses less risk, and operates under low speed. Under these conditions, the critical merging issue with ADS could be studied and findings from this study could be applied [to] other transportation systems designed to operate in a similar fashion. This study will use a new model for predicting braking events, speed and throughput when an AV accepts a critical headway to merge at a roundabout. This proposal will develop the modeling, estimation, and control framework needed to realize the safety and operational benefits by actuating the traffic through a small number of controlled AVs at a roundabout. Moreover, this proposal will help clarify what control strategies are effective at a roundabout as the AV penetration rate grows from zero to a few percent. The developed model would then be tested using a level 3 automation at two existing roundabouts from the Apponaug Circulator in Warwick, RI which would be equipped with sensors and cameras. Wireless magnetometers placed in pavements would be used to measure headway, speed and throughput during deployment. While as, installed cameras would be used to collect traffic video data which would be used in analyzing interaction between AVs, human drivers, and pedestrians to identify near-misses (scenarios that almost resulted in a collision) and generate heat-maps highlighting areas where near-misses were observed. Data collected from this project would be submitted to data repositories online which would be shared in real time.

FOCUS AREAS

With consistent advancement in driverless technology, AVs will soon form a larger part of our transportation system. The IEEE [12] predicts that AVs will account for up to 75% of vehicles on the road by the year 2040. One of the main benefits that is often promoted with AVs is safety, which is very true, but there are still some unanswered questions on how AVs can be integrated in existing transportation infrastructure without compromising its safety benefits.

One particular area that needs further attention is AV operation at locations such as roundabouts, merging, unprotected left turns and stop signs which require a significant amount of human input to navigate. This is a serious issue in the automotive industry as it is evident in the recent statement by Tesla CEO Elon Musk where the company made the public aware of its intention of testing new software for Tesla vehicles to safely navigate roundabouts and stop signs [13]. This is also evident in the ongoing Arizona pilot program launched by Waymo, Google's self-driving car unit, where human drivers had encountered that self-driving cars were having trouble merging onto freeways and also navigating unprotected left turns [14]. These issues that the launched pilot programs are experiencing can be examined with the use of modern roundabouts as they involve merging and gap acceptance. As with AVs, the modern roundabout has become an increasingly popular form of intersection control in the United States due to its effectiveness in improving safety and reducing traffic congestion. Since the first modern roundabout was built in Nevada in 1990, the number has increased significantly, and as of December 2018, more than four thousand have been constructed [15, 16].

A roundabout is an unsignalized circular intersection where the approaches are controlled by YIELD signs. At a roundabout, entering vehicles need to yield and enter the circle when there is a gap of acceptable length. This procedure of drivers scanning for gaps at roundabout to merge into circulating traffic is also used to merge onto freeways and to make unprotected left turns. If a model and testbed can be developed to study the effect of various critical gap acceptances of AV at roundabouts, then they can also be applied to other intersections designed to operate in a similar fashion and set the principles AV operation at intersections that require a significant amount of human-decision making to navigate. These principles could be used in creating new driving policy for safe operation of AVs at roundabouts and other intersections designed to operate in a similar fashion. Results from this study can also help the auto industry design better algorithms for AVs to safely operate at roundabouts and other merging situations. Also, with researchers and transportation practitioners in the US are currently exploring ways of improving existing roundabouts in case demand exceeds capacity [15], knowledge from this study will be important for the development of models that incorporates AV operation for use in improving existing roundabouts. Transportation professionals could also use this knowledge to design future roundabouts that could accommodate autonomous vehicle potential weaknesses and account for the safety of human traffic as well. In particular the project results will indicate needed side-road and on-board technology for safety.

The proposed demonstration satisfies the following Focus Areas outlined in the Notice of Funding Opportunity (NOFO) as follows:

A. Significant Public Benefits

Safety and Health Benefits: Efficient control of the bulk traffic will generate environmental and health benefits due to reduction in congestion, fuel consumption and emissions, and braking events. Knowledge on the critical headway acceptance behavior of AVs at roundabouts that results in less braking events is crucial in reducing crashes and enhancing vehicle throughput. It also reduces fuel consumption and emissions. Roundabouts on their own, are considered

sustainable intersections as they meet environmental, economical and socio-cultural sustainability requirements. As with roundabouts, AVs are also on track to soon form a larger part of our transportation system in the near future due to its safety and less fuel consumption benefits. This project provides the mathematical, computational, and engineering structure necessary for integrating human drivers with current and future civil infrastructure systems and possibly reap the environmental and economic benefits associated with these systems.

Sustainable Communities: The world's dependence on energy is increasingly devastating our planet. According to the International Energy Agency (IEA) estimates, global demand for energy is expected to increase by 80% by 2050. Several agencies are pursuing sustainable design measures to reduce the negative impacts on the environment. This project focuses on roundabout design and AV algorithms that could be designed to help transportation professionals respond to the emerging demand for sustainability. Roundabouts exhibit many of the characteristics of sustainable design, which include no power consumption by signal indicators and lower vehicle energy use and emissions. Roundabouts are also very effective in improving safety and reducing traffic congestion. Due to these benefits, roundabouts have become an increasingly popular form of intersection control in the United States.

B. Addressing Market Failure and Other Compelling Public Needs

AV Merging Operational Issues: Merging operations on roads have been a major problem among human drivers for a long time but were never got perfected; the problem is now affecting AV operations. For human drivers, there seems to be a general problem with finding the right time to merge, as some drivers merge too early while others merge too late. It seems cooperation among human drivers has been the key to safe merging maneuvers. This situation involves the use of hand gestures, eye contact, constant braking, accelerating, decelerating and good-judgment decision on the part of the driver. These human behaviors are often difficult for AVs to mimic thus making merging situations very challenging. This is evident in the ongoing Arizona pilot program launched by Waymo, Google's self-driving car unit, where human drivers had encountered the self-driving cars having trouble merging onto freeways and also navigating unprotected left turns [14]. A roundabout uses merging operation where entering vehicles need to yield and enter the circle when there is a gap of acceptable length. The geometric design of roundabouts encourages drivers to reduce speeds so as to be able to merge and circle the roundabout. Under these low speed conditions, different merging algorithms used by ADS will be examined in this demonstration together with developed dynamic controllers to detect various gaps in the traffic stream, congestion trends and event. Data will be collected with various commercially available vehicles, using sideroad and on-board technologies. We will also examine prescribed velocity controllers as part of this demonstration to investigate the safety and operational effects. Various models will be used for predicting braking event occurrences when an AV on the approach accepts a particular headway to merge into the circulatory pathway. The braking events for human drivers on the circulatory pathway would be assessed. The findings will help policy makers with developing and implementing new guidelines for the safe operation of AVs at roundabouts and other intersections designed to operate in a similar

fashion. With knowledge of the critical headway required for AVs to safely operate at roundabouts, government agencies can create standards for the auto industry to incorporate in AV technology. Under current conditions, where ADS testing is being performed by private companies and data are not accessible, it will be very difficult for government agencies to fully understand AV operation and develop policies for operation with human drivers. Data from this project will be readily available for developing and implementing new guidelines for the safe operation of AVs at roundabouts and other intersections designed to operate in a similar fashion.

C. Economic Vitality

Development of Intellectual Property: The proposed project provides new models, computational/software tools, and engineering solutions for assessing the safety performance at a roundabout. In addition, the control of traffic via moving actuators provides a new alternative to contemporary control technologies that can be applied in AV operations at highway merging/exiting, unprotected left turns and other intersections that require a significant amount of human-decision making to navigate. Currently, there is no United States guideline on how to improve an existing roundabout in case demand exceeds capacity, so most transportation professionals refer to studies conducted overseas that do not necessarily translate directly to domestic roundabout design and operation. The advent of ADS technology would further complicate the design and operation of roundabouts in the US. Roundabouts were invented for human drivers and requires direct driver input to navigate. Findings from this demonstration will provide US transportation professionals with a means of improving existing roundabout operational performance considering ADS technology as part of the transportation system. Also, knowledge from this demonstration could be used to design future roundabouts and other transportation systems that can accommodate autonomous vehicle weaknesses and account for the safety of human traffic as well.

Advancing Domestic Industry: The proposed study will retain Sensys Networks technology for sensors and camera installation, and data management. Sensys is a domestic company based in Berkeley, California. Sensys is the world's leading provider of integrated wireless traffic detection and data systems for Smart Cities. The company provides accurate and dependable detection data to drive reductions in urban traffic congestion for partners and public agencies around the globe. Sensys is currently partnering with UC Berkeley and Hyundai America Technical Center, Inc. (HATCI) as part of the NEXT-Generation Energy Technologies for Connected and Automated On-Road Vehicles (ARPA-E's NEXTCAR) Program to develop an innovative vehicle dynamics and power train control architecture based on a predictive and data-driven approach.

D. Complexity of Technology

High-level Automation: The proposed study will use high-level automation on vehicles and on roadside for demonstration purposes. The transition from stationary sensors to GPS data has proven to be a paradigm change in traffic estimation. The addition of ADS represents another paradigm change, with the potential to interweave data acquisition and control to an unprecedented extent. Since this project also provides also the mathematical and computational structure to address and employ these new connections, it will yield insight and ideas for the types of data acquisition techniques desirable in the future.

E. Diversity of Project

Variability in Services, Weather Conditions, and Driver Behavior: This project uses single and double-lane roundabouts that serve urban, suburban, rural and Native American communities in the state of Rhode Island. This would provide some insight on how developed algorithms would work for different road users. The testing would be performed for a period of two years capturing data in different weather conditions and seasons. In addition to the variability in site settings and seasons, driving behavior from different regions in the country would be incorporated during the software development and calibration phase as the PIs are based in the northeast, southwest and mountain plains parts of the US. Such behaviors will be obtained by driving commercially available vehicles equipped with onboard technologies in the states involved in this project.

F. Transportation-challenged Populations

Native Americans, Older and Disabled Drivers at Roundabout: Older and disabled drivers will be among those in the circulatory pathway of the roundabout, and these same drivers that would be used in this study during the software development and calibration phase. This study will use a model to examine the safety and operational requirements needed for autonomous vehicles to operate with older, disabled and regular human traffic at both single and double lane roundabouts. In addition, the research team will conduct user surveys of these transportation-challenged population groups to further evaluate their perspectives with regard to driver and pedestrian travel needs at roundabouts. This information will be used to feed and calibrate the operational models developed as part of this study.

G. Prototypes

Using AV to Test Control Algorithms at Roundabouts: In this study, an ADS will be tested at selected sites for a period of two years to examine safety effects of various critical headway acceptances of an AV on older, disabled; and regular human traffic drivers. Moreover, a complete testbed, including artificial and virtual environments will allows the creation of a multi-level testbed prototype for ADS testing in roundabout and complicated junctions.

REQUIREMENTS

The proposed demonstration would involve physically deploying vehicles with Level 3 automation at selected tech-equipped roundabouts in the state of Rhode Island to examine the safety and operational requirements needed for AVs to operate with older, disabled; and regular human traffic at both single and double lane roundabouts. Selected roundabouts would be equipped with cameras and sensors for capturing volume, occupancy and speed data. Data from on-board sensors mounted on vehicles would be captured as well. These real-time traffic data would be used in measuring accepted headway, monitoring of braking events of drivers, and measuring throughput. Project data would be submitted to data repositories online with password access which would be shared with the USDOT and other researchers. The testing of vehicles (without sideroad technology) will occur also in the states of Alabama, Delaware, Idaho, and New Jersey, providing a uniquely rich dataset which includes different road conditions, traffic behaviors and road infrastructures.

For this study, a multi-level testbed and models will be developed for predicting braking event occurrences when an ADS vehicle entering the roundabout accepts a particular headway gap to merge into the circulatory pathway. Headway and throughput data would be captured as well. The model would be scaled to replicate real-world traffic using varying settings, weather conditions, and driver behavior in different regions in the US in a mini-city environment. This would allow the model to be transferred to different parts of the country.

APPROACH

The project is based on creating a multi-level testbed for safety of ADS-equipped vehicles at roundabout. Moreover, data at roundabout for vehicles trajectories will be collected using sideroad technology (cameras and sensors) and on-board ones (OBD II). These data will be publicly available, stored in the Rutgers Amarel cluster, and used to fit the testbed tools.

Braking is a driver's option to avoid a collision. Predicting braking events is crucial in avoiding a collision. Smith et al. evaluated several test track car-following events, where the subject experienced several different crash imminent scenarios caused by the lead vehicle braking at different speeds and severities. The study found that, in the context of crash avoidance maneuvers, drivers generally initiate braking at longer distances than steering maneuvers in order to avoid a lead vehicle ahead in their lane of travel [16]. In this study, a model will be developed for predicting braking event occurrences when an AV on the approach accepts a particular headway to merge into the circulatory pathway. Headway and throughput data would be captured as well.

In order to examine the safety and operational effect of critical headway acceptances of AVs at roundabouts, appropriate mathematical abstractions are needed that capture both the bulk traffic flow, and the specific behavior of the low penetration rate AVs. We propose new micro-

macro traffic flow models to capture the important traffic quantities influencing braking events. Next, we will address how to estimate the bulk traffic flow using the new micro-macro models, and we pose determining the interaction rules between AVs and bulk flow as an estimation problem. With knowledge of the micro and macro traffic states, we will explore to what extent various controllers can be designed to safely merge into the circulating traffic, and at what sensor and actuator densities are needed to realize the benefits. Finally, verification techniques and demonstrations will be performed to allow the controllers to generate the desired outcomes both in simulation and on experimental cyber-physical system platforms.

To ensure we meet the program goals, focus areas, and demonstration requirements, specific project tasks are provided below.

Task 1: Equip both single and double-lane roundabouts with cameras and sensors

This task focuses on equipping selected single and double-lane roundabouts in Rhode Island with cameras and sensors. For analysis purposes, two existing roundabouts from the Apponaug Circulator in Warwick, RI were chosen.

This choice will be further analyzed and discussed after traffic data collection. In particular other sites will be considered presenting roundabout in the same area. The final decision will be taken according to the results of data collected from RI DOT, in terms of daily traffic load, number of accidents and other available data.

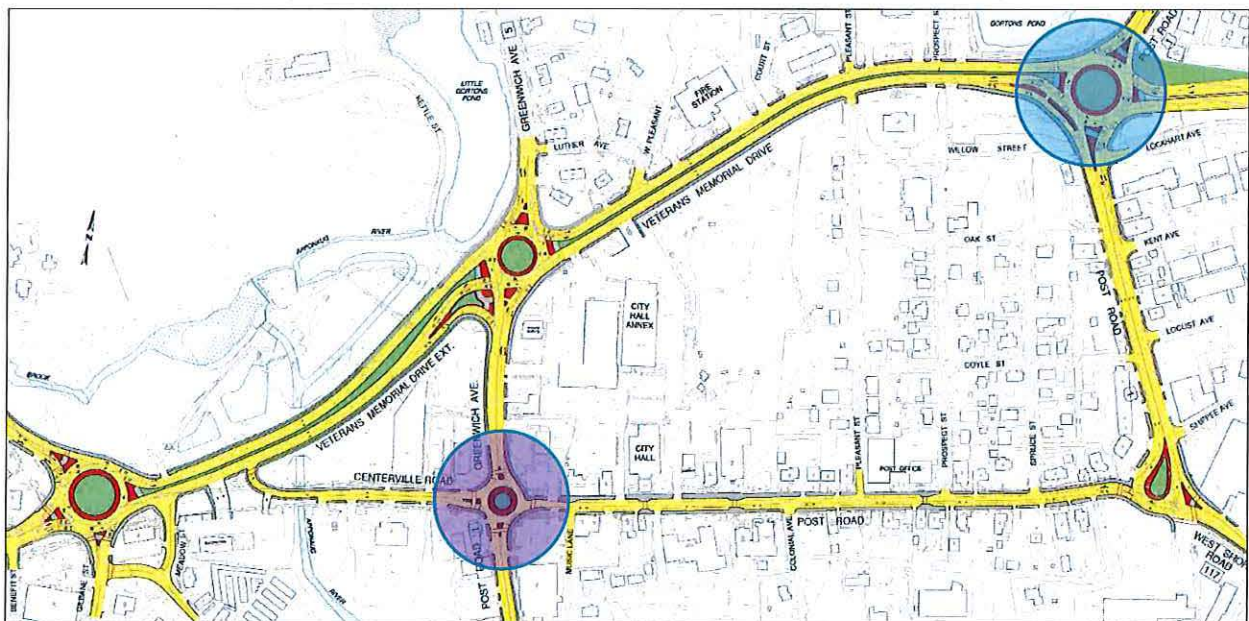


Figure 1: Apponaug Circulator, Study Sites Highlighted (Source: RIDOT)

Apponaug Circulator

The Apponaug Circulator is a five-circulator system consisting of five roundabouts in the historical village center of Apponaug. The project was undertaken by the Rhode Island Department of Transportation to increase driving safety by eliminating the possibility for T-bone crashes and lower speeds while accommodating a 30-50 percent increase in traffic capacity [17].

The Apponaug Circulator is more pedestrian and bike friendly, accommodates people with disabilities, and serves several restaurants and shops located in the vicinity. Apponaug has a rich Native American heritage associated with the Narragansett Tribe and now sits in the center of Warwick, which is the third largest city and is located in the center of Rhode Island. The Apponaug Circulator serves approximately 6,800 vehicles daily which come from different urban, suburban, rural and Native American communities in the state.

The roundabouts (highlighted in Figure 1) chosen for this study are:

- Double Lane Roundabout located at Veterans Memorial Drive and Post Road (highlighted in blue in Figure 1)
- Single Lane Roundabout located at Centerville Road and Greenwich Road (highlighted in purple in Figure 1)

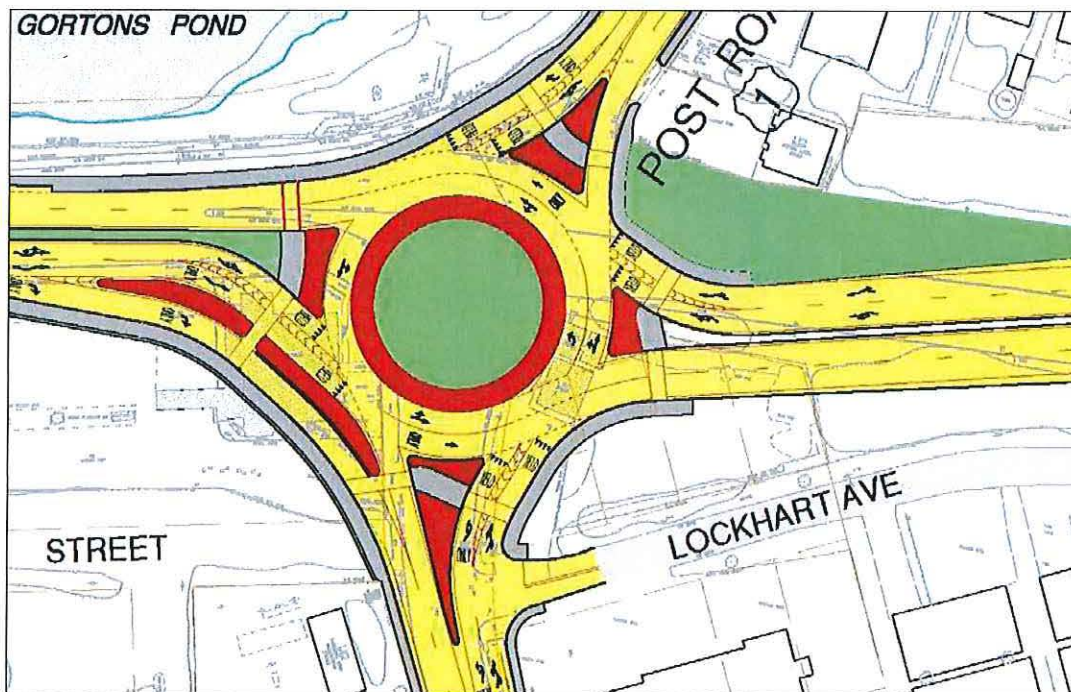


Figure 2: Double Lane Roundabout at Veterans Memorial Drive and Post Road (Source: RIDOT)

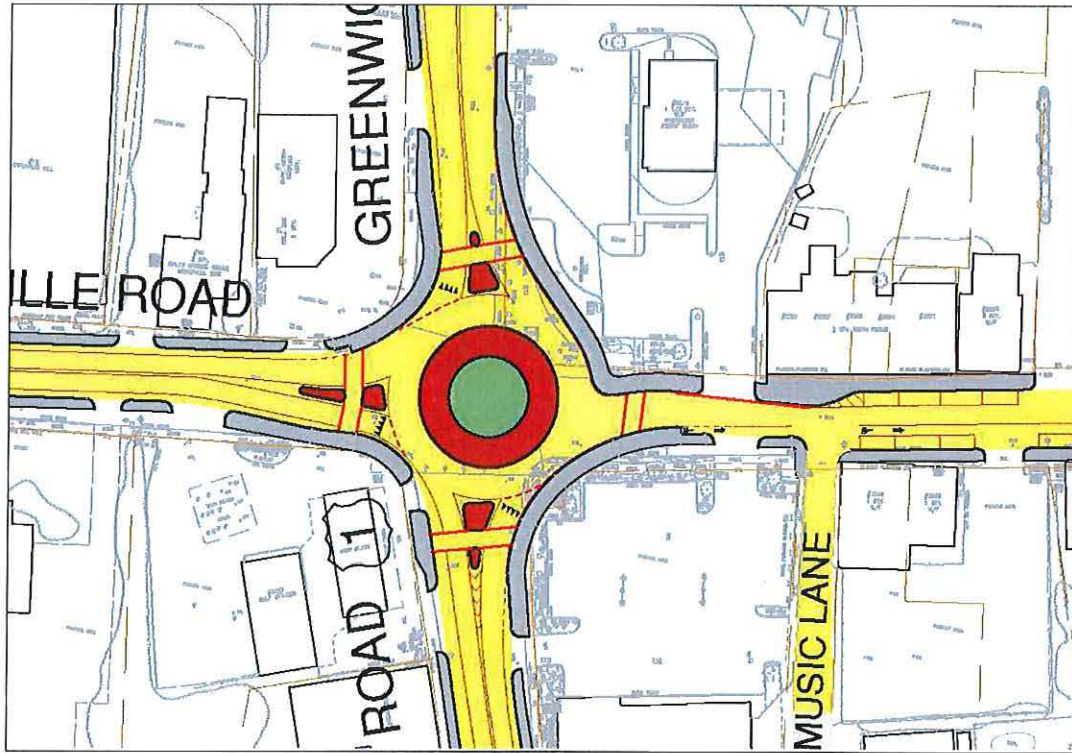


Figure 3: Single-Lane Roundabout at Centerville Road and Greenwich Road (Source: RIDOT)

Both roundabouts have physical configuration comparable to typical single and double-lane roundabouts in the US which allows demonstration data to be conveniently transferred to applicable environments. Both have four legs, minimum entry width of 14 feet and circulatory width of 15 feet. The double-lane roundabout has an inscribed circle diameter of 170 feet and the single-lane roundabout has an inscribed circle diameter of 120 approximately. They also have a balanced proportion of elderly, disabled and regular human drivers.

Measuring Traffic Stream Parameters during Demonstration

To automate the data collection process we will use multiple traffic detectors. Such devices include loop detectors, radar, and other forms of vehicle detection. In particular, dual-loop detectors will be used to measure speed, headway, and other parameters. A specific technology that has been successfully used to measure traffic stream parameters at roundabouts is wireless magnetometers. Wireless magnetometers are an in-pavement sensor that detects the presence or occupancy of a vehicle or metal object. These devices are small and can be easily installed in about 5-10 minutes. With volume data, occupancy data, and other observations, much information about traffic movements will be collected. Hainen et al. [18] reported on the application of wireless magnetometers to collect point presence detection for calculating the rejected critical headways of a single lane roundabout at Spring Mill Road and West 106th Street in Carmel, IN. The roundabout was instrumented with 16 wireless magnetometers in the entering, exiting, and circulating lanes on all four legs of the intersection.

For this project, Sensys Networks would be retained to install wireless magnetometer point sensors at selected roundabouts to collect data during demonstration. Data collected will be used in calculating accepted headway of the AV, throughput and speed. In addition to the wireless magnetometers, video cameras would be installed at the selected roundabout sites for continuous real-time conflict detection. Video analytics software would be used for continuous deep learning analytics on collected traffic video. Using deep vision analytics on traffic video, the software would manage cameras and an automated video data collection, monitoring and analysis platform to observe the interaction between AVs, human drivers, and pedestrians to identify near-misses (scenarios that almost result in a collision). Near-miss collision analysis will enable the prediction of safety incidents before they happen and also be used for qualitative comparative analysis with developed traffic models.

A more detailed plan for data collection, storage and sharing would be developed under this task once roundabout sites are selected. In general, all in-ground and out-of-ground events data would be transmitted locally to an access point and then to SQL database over cellular communication networks. Event data would automatically be processed to yield per-vehicle or per-lane statistics. Predefined and customized reports would present accepted headway, speed and throughput data in a web-based format, which will be shared publicly.

Task\Partner	Rutgers	Alabama	Delaware	Idaho	RI	Sensys
Task 1.1 Site selections	20%	40%			40%	
Task 1.2 Data collection plan	30%	40%			30%	
Task 1.3 Sensor deployment and data collection						100%
Task 1.4 Data storage	70%	30%				

Deliverable 1.1: Sensors and cameras installed per layout plan and a criteria for capturing and storing data.

Deliverable 1.2: Storage of data collected by camera and sensors

Task 2: Develop a software suite including microscopic and macroscopic models calibrated for the sites

Traffic models range from microscopic (describing every single car position and speed) to macroscopic (describing the network flows), see [19,20]. For the project we will develop a software suite based on multiple and integrated scales:

1. Microscopic models can be a written system of ordinary differential equations such as $\dot{x} = v$, $\dot{v} = f(x, v)$, where x represents the position and v the speed of the vehicle. This class of systems include the classical Gipps model (for discrete time steps), Follow-the-

Leader, and the Bando or Optimal Velocity. Such models are also at the base of available commercial software such as Sumo, Aimsun, and others. Moreover, cellular automata can be seen as time-discretization of these models.

2. Mesoscopic models, which describe the probability distribution of positions and speeds along a stretch of road, e.g. the Prigogine model.
3. Macroscopic, which describe traffic at the level of aggregated variables, such as mean velocity, density and flows. The celebrated Lighthill-Whitham-Richards models belong to this class. Moreover, cell transmission models are time-discretization of such models. Macroscopic models were already developed by the team for roundabouts in [21], see Figure 4 for an example of a two-lane roundabout.
4. Mean-field games and multi-scale models. These models aggregate microscopic models to obtain macroscopic ones but also considers control policies for the single driver and allows for the consideration of mixed scales with different car groups (e.g. AVs) described at the microscopic level opposed to bulk traffic described at macroscopic level.

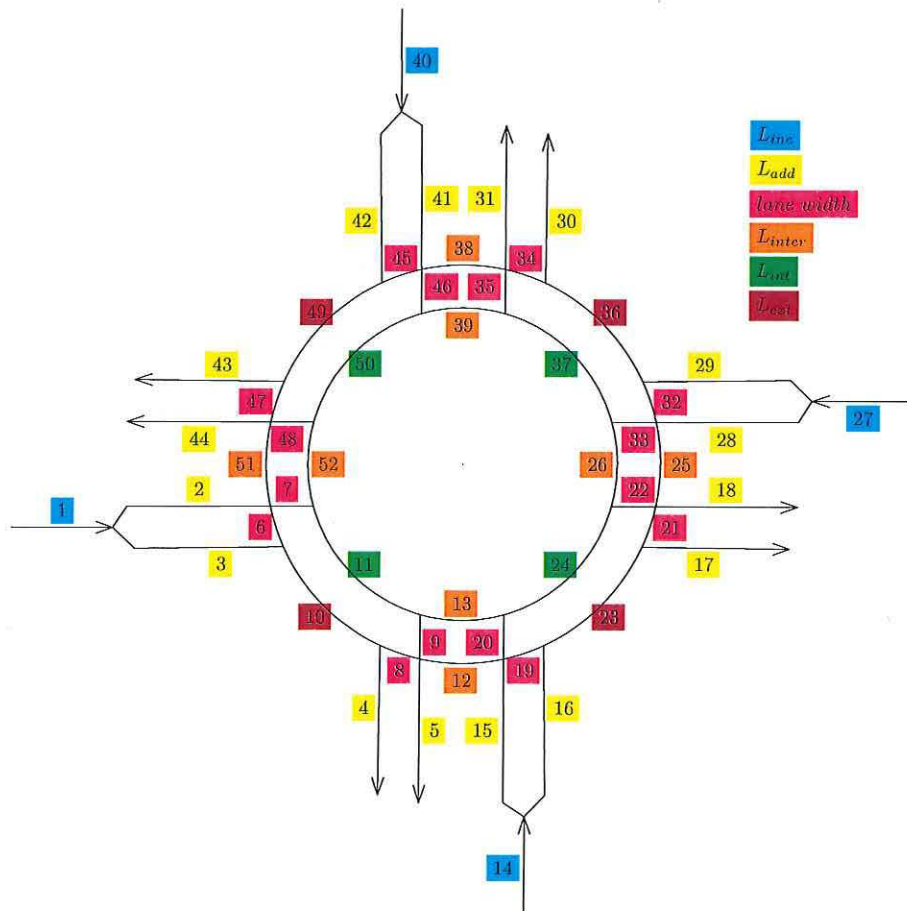


Figure 4: A road network representing a roundabout from [21]. Each colored label represents a different type of meeting point (potential conflictual car trajectories). Different colors correspond to different type of meeting points.

To capture the effect of AVs and ADAS immersed in bulk traffic at roundabouts we will use multi-scale models, obtained via a mean-field limit approach. We will proceed as follows:

- 1) Design and tune microscopic models for the selected sites.
- 2) Design and tune macroscopic models for the selected sites.
- 3) Use mean-field approaches to merge scales and generate a single software simulating the different scales: microscopic for the AVs and ADAS-equipped cars and macroscopic for bulk traffic.

To illustrate the macroscopic approach, consider a roundabout, which can be seen as a sequence of junctions and represented visually in which roads are described by arcs and junctions by vertices (see Figure 4). The roundabout can be equipped with 52 links of which 16 belong to entrances, 12 belong to exits, 16 belong the outer lane of the circle and 8 belong to the inner lane of the circle.

Task\Partner	Rutgers	Alabama	Delaware	Idaho	RI
Task 2.1 Design and tune of microscopic models	40%	60%			
Task 2.2 Design and tune of microscopic models	60%	40%			
Task 2.3 Design and tune of mean-field models	80%	20%			
Task 2.4 Model simulations	80%	20%			

Deliverable 2.1: Software suite for tuned to selected sites.

Deliverable 2.2: Simulation results for roundabouts with one or two lanes.

Task 3: Extend and calibrate the mini-city environment for the sites, including matching software outcomes

Under this task, the selected sites would be modeled in a scaled test-bed that replicates real-world traffic at the selected sites. The locations of sensors placed in task 1 would be used to set up the testbed. Using this testbed, we can quickly, safely, and affordably experimentally calibrated control concepts aimed at enhancing our understanding of the safety implications of an AV accepting critical headway on an approach of a roundabout.

The University of Delaware Scaled Smart City (UDSSC) is a 1:25 scaled test-bed spanning over 400 square feet (Figure 5) and can accommodate at least 35 scaled connected and automated vehicles (CAVs). It is equipped with a VICON motion capture system which uses eight cameras to track the position of each vehicle with sub-millimeter accuracy. Each road in the UDSSC is built from arc or line segments; to track the desired vector position of each CAV, all road segments are parameterized in terms of their total length. This formulation

allows each vehicle to calculate its desired position in UDSSC based only on the scalar distance along its current path, which is achieved by numerically integrating its speed profile in real-time on both the mainframe computer and the CAV. This decoupling of speed and position allows significant flexibility in UDSSC, especially in dynamic-routing scenarios.

UDSSC can replicate real-world traffic scenarios and implement cutting-edge control technologies in a safe and scaled environment. UDSSC is a fully integrated smart city, which can be used to validate the efficiency of algorithms and their applicability in hardware. It utilizes high-end computers, a VICON motion capture system, and scaled CAVs to simulate a variety of control strategies with as many as scaled CAVs.

Coordination of CAVs within UDSSC is achieved using a multi-level control framework spanning a central mainframe computer (Processor: Intel Core i7-6950X CPU @ 3.00 GHz x 20, Memory: 125.8 Gb) and the individual CAVs in the experiment (Raspberry Pi 3B). The mainframe runs an Ubuntu 16.04.5 LTS Linux distribution and ROS Kinetic. High level routing is achieved by a multithreaded C++ program running on the mainframe computer. At the start of the experiment each CAV sets its temporal baseline from which it measures all later times; this avoids the problem of synchronizing CAV clocks as all information is calculated relative to the experiment start. During the experiment the mainframe passes messages to each CAV containing its current position and two seconds of trajectory data in UDP/IP format at 50 Hz. The CAV receives trajectory information from the mainframe and uses a nonlinear steering controller and a feedforward-feedback position controller to navigate the city.



Figure 5: A birds-eye view of the University of Delaware's Scaled Smart City.

Scaled Connected and Automated Vehicles

The CAVs used in the UDSSC, presented in Figure 5, have been designed using off the shelf electrical components and 3D printed parts created at the University of Delaware. The primary microcontroller on the CAV is a Raspberry Pi 3B running Ubuntu Mate and ROS

Kinetic. An Arduino Nano is used as a slave processor for Pi to do a low-level motor control and ad-hoc analog to digital. The CAV's rear-wheel drivetrain is powered by a pololu 75.8:1, 6 V micro metal gearmotor; the motor is controlled using a motor controller, and encoder for feedback, hooked into the Arduino for low-level speed and position control. Power from the gearmotor is transferred to the rear axle with two 3D printed gears with a 1:1 ratio. Two rubberized wheels with a 1.6 centimeter radius are mounted directly to the rear axle. The motor controller receives power through a 5 V regulator, and a pulse-width modulated command from the Arduino is used to control the motor's speed. Steering is achieved by a custom 3D printed Ackermann-style steering mechanism actuated by a Miuzei micro servo motor, which again is controlled directly by the Arduino. The CAVs are also equipped with a Pi Camera, ultrasonic sensors, and a voltage measurement circuit to collect experimental data and reduce the overall reliance on VICON. A power regulator manages the voltage requirement of the Pi and Arduino by supplying the regulated 5 V DC power from two 3000 mAh, 3.7 V Li-ion batteries configured in series. With this hardware configuration, the CAV is able to run and collect experimental data at 20 Hz for up to 2 hours.



Figure 6: Vehicles used in the scaled smart city with internals (left) and outer shell (right).

Task\Partner	Rutgers	Alabama	Delaware	Idaho	RI
Task 3.1 Replication of selected sited in mini-city	20%	20%	60%		
Task 3.2 Testing of human-driven mini-cars		10%	90%		
Task 3.3 Testing of Arduino control algorithms	10%		90%		

Deliverable 3.1: Mini-city replication of selected sites with one-lane and two-lanes roundabout.

Deliverable 3.2: Mini-city test data for human-driven and Arduino control algorithms mini-cars.

Task 4: Design control algorithms for braking events and congestion reduction using pipeline of tasks 1-3 during merging operation

With knowledge from tasks 1-3, design control algorithms for reduced braking events would be designed for the microscopic and multi-scale models.

Our strategy for this task will be as follows:

- 1) Use Task 2 microscopic models tuned to selected sites for parameters identification.
- 2) Formulate optimal control problems for AVs and ADAS-equipped cars at roundabouts to reduce or eliminate braking events.
- 3) Solve control problems with optimization software and deep-learning approaches.
- 4) Test the control algorithms on the microscopic and multi-scale models of Task 2.

Recently it was shown that a small number of AVs immersed in bulk traffic are able to dissipate stop-and-go waves [22]. Building on this experience we propose a control algorithm for AVs and ADAS-equipped car to improve traffic flow at roundabouts. The experiments reported in [22] showed that a single autonomous vehicle (representing an AV penetration rate of approximately 5%) carefully controlled was able to eliminate stop-and-go waves on a ring-road track reducing the total fuel consumption of up to 40% and almost eliminate braking events (see Figure 7).

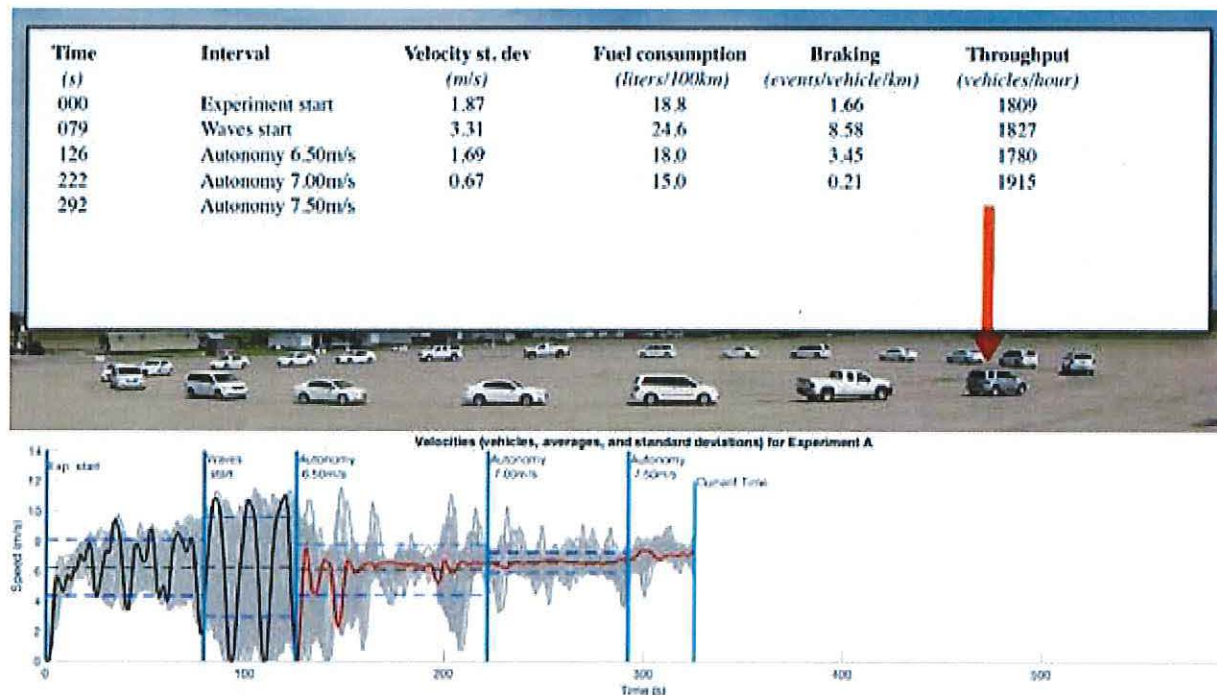


Figure 7: Controlled AV eliminating traffic waves and braking events from Youtube video [23]. The experiment was performed on a ring-road traffic with a single AV immersed in a fleet of 20 cars and resulted in fuel consumption reduction of up to 40% and braking events reduction of up to 98%.

The ring-road track will represent the internal part of the single-lane roundabouts targeted in this project. Notice that the control algorithms will aim not only at entering safely the roundabout, but also at minimizing the impact on the traffic flow. Controls will be designed also for the ADS-equipped and AVs already in the roundabout internal ring so to optimize fuel consumption and minimize breaking events of all the cars in the roundabout.

The impact may extend to emissions (see [24] for results on the ring-road experiments), but this project will only estimate this impact via 'Modal' type models. While such models are not reliable to estimate a precise fuel consumption of a single car during, they can be effectively used to calculate the expected benefit in terms of air quality when the overall traffic is smoothed by the AVs actions.

Task\Partner	Rutgers	Alabama	Delaware	Idaho	RI
Task 4.1 Formulate control problems	50%	20%	30%		
Task 4.2 Use optimization tools for numerical design	60%	20%	20%		
Task 4.3 Test control algorithms	90%	10%			

Deliverable 4.1: Matlab code for numerical optimization of ADS velocity profiles at roundabout.

Deliverable 4.2: Simulation results for control algorithms with multi-scale models.

Task 5: Use AVs and ADS-equipped cars to test control algorithms and currently available technology at roundabouts and junctions in five states

Under this task, the ADS-equipped cars and AVs would be tested at the selected site using car manufacturer technology (auto-pilot, ACC, safety signals, super cruise and other) and additional on-board technology (OBD II, Lidar, Mobileye) for a period of two years with planned testing during each season.

The testing will occur in all five states: for Rhode Island on the selected site (see Task 1) with roadside sensors and cameras, while in other states at sites to be selected. The sites targeted will consist mostly of roundabouts but other type of junctions will be considered, in particular 4-way stop ones. Users surveys will anticipate the tests.

Then three cars (Tesla Y, Cadillac with Super Cruise and Toyota Camry) will be driven 100 days in RI and 100 days in the other states (cumulative) for a grand total of 5,000 driving hours and 100,000 miles of data.

We will also eventually leverage resources from ARPAE's NEXTCAR project led by Professor Andreas Malikopoulos, in particular an Audi PHEV, see description below.

Summarizing the approach will consist of the following data gathering modality:

- 1) Data on human driving at roundabout. Using side-road and on-board technology we will provide extensive data on driving behavior in different locations and seasons.
- 2) Data on currently available technology from car manufacturer and tech provider. This will include ACC, auto-pilot and other common devices.
- 3) Data on experimental vehicles such as the Audi PHEV.

Data will be collected during testing, and managed and stored at Rutgers Amarel cluster as outlined in detailed data management plan. The wireless magnetometers would be used to measure headway, speed and throughput during deployment. Installed cameras would be used to collect traffic data which would be used for qualitative comparative analysis with developed traffic and braking event models. Models developed in Tasks 3 and 4 would be modified as needed.

Using video analytics, the speed profile for each vehicle in the circulatory path could be tracked for changes in speed pattern when an AV enters the circle with a specified headway which would be measured with the magnetometers and the video data. Collected video data would be used in analyzing interaction between AVs, human drivers, and pedestrians to identify near-misses (scenarios that almost resulted in a collision) and generate heat-maps highlighting areas where near-misses were observed.

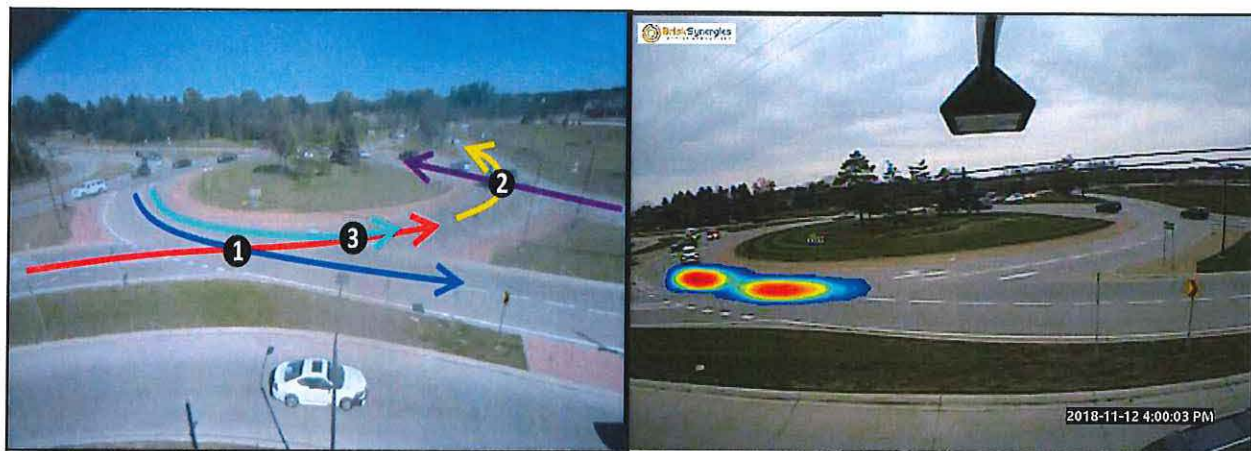


Figure 8: Speed profile (left) and conflict heat-map (right) (Source: BriskSynergies [23])

Since autonomous vehicles will be sparse, important theoretical and practical issues must be considered in terms of their effectiveness. Under what circumstances can algorithms be proven to work, and with what margins? At what rates must data be available, and do these rates depend on the sparsity of sensor and actuator vehicles? Each phenomenon may have different criteria, but these are critical to understand at a fundamental level. For instance it was proven that a penetration rate of 5% is sufficient for monitoring or even control purposed (see [22]), but experimental results for roundabouts are not yet available.

At a practical level, we must ensure that an available technology or a controller, when executing, is not making the phenomenon worse. For such scenarios, we plan to take advantage of our expertise in code generation techniques to embed proofs of correctness at runtime. If a controller is making the situation worse, it may need to be discontinued in favor of a passive behavior. The testing will be performed both on car-manufacturer technology and on control algorithm developed in Task 4 and communicated to drivers for a manned execution. The demonstration to be performed here is in expressing the dynamic results that indicate improved performance and embedding those guarantees into the controller code. This effort is not as simple as responding to an "if-then-else condition", as controller switching must obey continuous system constraints on safety margins. The results will be based on existing work in software verification for switched autonomous systems [26, 27, 28].

Data collected would be used in predicting breaking events, calculating headway, throughput and speed. The experiments will be anticipated by hardware-in-the-loop simulations (using tools like Carla and Gazebo) to simulate the detection of traffic events, using logical vehicles and sensors, and analyze the necessary stability criteria for the traffic controllers. The code generators from previous projects [21, 22, 24], as well as some tools developed from this project, will be used to integrate simulation environments to provide these proofs of concept. The research team has significant expertise in heterogeneous simulation and integration of cyber-physical systems simulation.



Figure 9: The demo vehicle Audi A3 e-tron.

Audi A3 etron

The project can eventually leverage resources and knowledge developed through the ARPAC's NEXTCAR project led by Professor Andreas Malikopoulos. The overarching goal of this project is to develop and implement control technologies aimed at maximizing the energy efficiency of a 2016 Audi A3 e-tron plug-in hybrid (Figure 9) electric vehicle (PHEV) without degradation in tailpipe out exhaust emission levels, and without sacrificing the vehicle's drivability, performance, and safety. These technologies will exploit connectivity between vehicles and the infrastructure to optimize concurrently vehicle-level and powertrain-level operations through a two-level control architecture to: (1) optimize the vehicle's speed profile aimed at minimizing (ideally, eliminating) stop-and-go driving, and (2) optimize the powertrain of the vehicle for this optimal speed profile obtained under (1). The team will be able to use the Audi for some experimental validation of the tasks on the round about to be considered.

Task\Partner	Rutgers	Alabama	Delaware	Idaho	RI
Task 5.1 Site selection and surveys administration	30%	20%	10%	30%	10%
Task 5.2 Driving tests and data collection	20%	20%	20%	20%	20%
Task 5.3 Data reconciliation and storage	40%			60%	

Task 6: Management

This task will be led by the Rutgers team, but all the participating units, in particular PIs and students, will be consulted and involved in the activities. Moreover, this task will include the management of the relationships with US DOT.

The project manager will be hired by Rutgers and will coordinate the Leadership Team composed of the manager and the participating PIs. The leadership team will meet at the beginning of the project for a kick-off and at various milestones as will be agreed with US DOT. Moreover, the Leadership Team will organize regular (at least monthly) virtual meeting via Zoom (or other conference software) to discuss project advancements, deliverables and reporting. The Leadership Team will also benefit of the logistic and administrative support of Rutgers University and the other participating institutions.

The project manager will consult regularly with the main PI (Piccoli) to coordinate activities, making sure the project progresses according to the established timeline, organize in-person and virtual meeting with project partners and US DOT as needed. The project manager will be supported by the PIs for deliverable preparation, documentation and reporting to US DOT. The project manager will also supervise the interaction with the companies involved in the project for material and data acquisition.

The project manager will also supervise the data storage process together with the involved PIs, their dissemination and making sure the availability meets the US DOT established criteria.

Task\Partner	Rutgers	Alabama	Delaware	Idaho	RI
Task 6.1 Activities coordination	70%	10%	10%		10%
Task 6.2 Deliverable preparation and standardization	70%	10%	10%	10%	
Task 6.3 Documents and reporting	70%	10%		10%	10%

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